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# FUZZY IDENTIFICATION OF ENERGY-EXCHANGE MODELS FROM TECHNOLOGICAL PROCESSES


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An identification method is proposed that enables one to estimate the parameters and also to evaluate the model performance. The method is based on fuzzy-set theory.


Very often, there are several competing models available to simulate a complex phenomenon or process, and these contain adapted parameters that require experimental identification.

Leaving aside aspects such as the computer run time and the algorithms involved, we can say that the best model is selected on criteria for accuracy and physical acceptability in the values obtained for the adapted parameters. The estimates of accuracy and physical acceptability are dependent on fuzzy factors involved in the subjective preferences of those developing and using the model, so it is desirable to use the theory of fuzzy sets to formalize the choice of the optimum model [1].

This analysis is made with reference to simulating the thermal and energy processes in the hot rolling of aluminum alloys. The following form can be given for the basic model for the processes in the rolling cage, which is represented by a system of nonlinear algebraic equations [2]:

$$T_1 = f(\sigma, \ \alpha, \ P, \ T_0, \ H_0, \ H_1, \ v), \tag{1}$$


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$$P = g\left(\sigma, \frac{T_1 + T_2}{2}, H_0, H_1, v\right). \quad (2)$$

We compare the model for calculating the force on the roll based on generalization from the experimental data:

$$P = 2L_D Bk\left(1.5 \ln\frac{1}{M} + 1.5M - 0.5\right), \quad M = \frac{L_D}{H_{av}} < 1, \quad (3)$$

$$P = 2L_D Bk\left(\ln M + \frac{1}{M}\right), \quad M \geqslant 1, \quad (4)$$

and Sim's theoretical model [3]:

$$P = 2L_D Bk\left(\frac{\pi}{2}\sqrt{X}\operatorname{arctg}\sqrt{\frac{1}{X}} - \frac{\pi}{4} - X\sqrt{\frac{R}{H_1}}\ln\frac{H_{av}}{H_1} + \frac{1}{2}X\sqrt{\frac{R}{H_1}}\ln\frac{X+1}{X}\right), \quad X = \frac{H_1}{H_0 - H_1}. \quad (5)$$

The dependence of the roll force on temperature is incorporated via the deformation resistance $k = \sigma f_1(T) f_2(H_1, H_0) f_3(v)$; the adapted parameters in the models of (1), (3), (4) and (1), (5) are the heat-transfer coefficient $\alpha$ from the strip to the roll and the deformation resistance $\sigma$ for particular alloys under standard conditions.

In the experiments, one measures the force $P_e$ on the rolls and the temperature of the strip in the last cage $T_e$.

The accuracy criteria are specified in the form of a function in which the calculated values $P_c$ and $T_c$ belong to the range of values in which they correspond closely to the experimental data $C_1(P_e - P_c)$, $C_2(T_e - T_c)$; the functions $C_1$ and $C_2$ take maximum values of one for $|P_e - P_c| \leqslant \delta_1$, $|T_e - T_c| \leqslant \delta_2$, correspondingly, and monotonically decrease to zero as the calculated values deviate from experiment. Here $C_1$ is unsymmetrical because for technological reasons it is preferable to obtain overestimates for the calculated forces on the rolls rather than underestimates. The functions $C_3(\alpha)$, $C_4(\sigma)$ for the relation to the physically permissible range are also specified as values normalized to unity on the basis of published data. If there is no information on the degrees of preference for the various values of the arguments in $C_1$, $C_2$, $C_3$, and $C_4$ within the permissible ranges, then these fuzzy criteria degenerate into ordinary constraints of inequality type.

For each point from experiment $P_{ei}$, $T_{ei}$, the intersection of the criteria for accuracy and physical feasibility of the model

$$D_i = [C_1(P_{ei} - P_c)]^{\alpha_1} \cap [C_2(T_{ei} - T_c)]^{\alpha_2} \cap [C_3(\alpha)]^{\alpha_3} \cap [C_4(\sigma)]^{\alpha_4} \quad (6)$$

will be dependent only on the adapted parameters $\alpha$ and $\sigma$. Following [3], for the intersection $\cap$ we use the operation of taking the minimum, while the absolute ranks of the criteria $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$ are calculated from the criterion pair-comparison matrix.

Clearly, an ideal and absolutely correct model should maximally satisfy all criteria for each experimental point. Therefore, $D_{i,u,\max} = \max_{\alpha,\sigma}[D_{i,u}(\alpha, \sigma)] = 1$ for an ideal model for any experimental point i. In that case, $D_{i,\max}$ for a real model can be considered as a measure of the closeness of the model to the ideal one or as a measure of the model performance for the given experimental point.

The overall performance measure for a set of N experimental points can be taken as the average estimator for the performance measure:

$$D_{av} = \frac{1}{N}\sum_{i=1}^{N} D_{i\max}. \quad (7)$$

Clearly, the optimum model will be the one that provides the largest value of $D_{av}$.

This method enables one to estimate the adapted parameters while choosing the optimum model. Let $D_{i\,max}$ be attained on a given set of experimental points together with the corresponding $\alpha_i$ and $\sigma_i$ providing maximum values for the $D_i$. The value of $D_{i\,max}$ can be interpreted as the degree to which experiment point i belongs to the region of adequate operation. Then the parameters are estimated from the formulas

$$\alpha = \sum_{i=1}^{N} \alpha_i D_{i\max} \Big/ \sum_{i=1}^{N} D_{i\max}, \tag{8}$$

$$\sigma = \sum_{i=1}^{N} \sigma_i D_{i\max} \Big/ \sum_{i=1}^{N} D_{i\max}. \tag{9}$$

As a result, the parameters are estimated on the basis of the performance or degree of suitability for the various experimental points in the model identification, and one uses the same information as in estimating the model performance.

Clearly, the results from fuzzy identification will be dependent on the form of the criterion. A global criterion of the form of (6) can be used with various ranking coefficients. Here it is possible that in some experiments it proved impossible to make complete measurements of all the output variables, for example, one measures only $P_e$ or $T_e$ or the experiments were performed at different times and with different accuracies.

For all these cases one can construct the corresponding global criteria $D_1$, $D_2$, $D_3$, etc. The person who makes the decision in choosing the model always has some concepts on the best form for the global criterion, so each of the $D_1$, $D_2$, $D_3$ can be put into correspondence with the value of the function $\varphi(D)$ representing the relation of this criterion to the hypothetical best one [4]. As a result, the ideal performance criterion D may be represented as a fuzzy subset. For example, for the three criteria,

$$D = \left\{ \frac{\varphi(D_1)}{D_1}, \frac{\varphi(D_2)}{D_2}, \frac{\varphi(D_3)}{D_3} \right\}. \tag{10}$$

On the other hand, the values of the criteria $D_1$, $D_2$, $D_3$ themselves can be considered as fuzzy functions for the models belonging to the ideal ones. For example, if one compares two models $Y_1$ and $Y_2$, then (10) is put as

$$D = \left\{ \frac{\varphi(D_1)}{\left\{\frac{D_1(Y_1)}{Y_1}, \frac{D_1(Y_2)}{Y_2}\right\}}, \frac{\varphi(D_2)}{\left\{\frac{D_2(Y_1)}{Y_1}, \frac{D_2(Y_2)}{Y_2}\right\}}, \frac{\varphi(D_3)}{\left\{\frac{D_3(Y_1)}{Y_1}, \frac{D_3(Y_2)}{Y_2}\right\}} \right\}. \tag{11}$$

Here the solution amounts to defining the model $Y_1$ or $Y_2$ having the maximum assignment function in D. Here $D_1(Y_1)$, $D_1(Y_2)$, $D_2(Y_1)$ and so on are taken as means on the set of experimental points.

In the above example, the rolling-model performance was estimated from data obtained with a continuous-running hot-rolling mill for three aluminum alloys. In all cases, the forces and temperatures were measured simultaneously on the last cage.

The measurements in all the experiments were made with the same accuracy, so the task of fuzzy identification was to maximize the function of (6) for each point with respect to α and $\sigma_i$, where $P_c$ and $T_c$ were calculated either from the model of (1), (3), and (4) or from the model of (1) and (5); (8) and (9) were used to calculate $D_{av}$ for each of the models and to estimate $\alpha$ and $\sigma$. The maximization was performed by a modified form of the Mark-Quardt method and by successive quadratic approximation.

The alloys A1, D16, and AMg6 were used in the experiments, and for each alloy we took 10 experimental points differing in rolling scheme. The model performance was evaluated by averaging over all 30 experimental points, while the parameters were estimated for each alloy separately, since they have different $\sigma$ under standard conditions. It was found that the best model was Sims's one with $D_{av} = 0.81$, with a somewhat lower value $D_{av} = 0.73$ for the criterial model. The estimates of $\sigma$ for the various alloys were within the limits of the natural spread for aluminum alloys, while the estimated values of $\alpha$ for the various alloys differed by not more than 10%. As these models are insensitive to changes in $\alpha$, one can take $\alpha$ as the same for all the alloys, which does not conflict with the physics of the process. The studies confirm that this method is effective in identification with simultaneous estimation of model performance with fuzzy criteria.

## NOTATION

$T_1$, $T_2$, inlet and outlet cage temperatures; P, roll force; $H_0$, $H_1$, strip thickness at the inlet and outlet; v, rolling speed; $L_D$, grip arc length; B, strip width; $H_{av}$, mean thickness of the deformation site; R, roll radius; $P_e$, $T_e$, measured roll force and temperature behind the cage; $P_c$, $T_c$, calculated roll force and temperature.

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